

The Higgs Connection – Flavor and Dark Matter

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Three families of quarks and leptons,
one Higgs to rule them all,
and in the darkness bind them.

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With the recent discovery [1, 2] of the 126 GeV particle at the Large Hadron Collider (LHC), and the likelihood of it being the one physical neutral Higgs boson h of the standard model (SM) of quarks and leptons, it may be a good time to reflect on what it means as to some of the other outstanding problems in particle physics. In this talk, I focus on flavor and dark matter (DM). I will show in eight easy steps how the one Higgs boson may be the key to understanding flavor through its interaction with dark matter [3].

Step 1: In addition to the observable sector of SM particles, let there be a dark sector, odd under Z_2 which may be derived from an $U(1)_D$ gauge symmetry. The particles of this dark sector consist of three neutral singlet Dirac fermions $N_{1,2,3}$ and the scalars

$$(\eta^+, \eta^0), \zeta^{-1/3} \sim \underline{5}, \quad (\xi^{2/3}, \xi^{-1/3}), (\zeta^{2/3})^*, \chi^+ \sim \underline{10}, \quad (1)$$

which are complete $SU(5)$ multiplets. The lightest N is stable and a possible candidate for the observed dark matter of the Universe.

Step 2: A non-Abelian discrete symmetry is imposed, under which $N_{1,2,3}$ as well as the three families of quarks and leptons of the SM transform nontrivially. As a concrete example, consider the non-Abelian discrete symmetry A_4 [4, 5, 6, 7], which is also the symmetry group of the tetrahedron. It has four irreducible representations $\underline{1}, \underline{1}', \underline{1}'', \underline{3}$, with the multiplication rule

$$\underline{3} \times \underline{3} = \underline{1} + \underline{1}' + \underline{1}'' + \underline{3} + \underline{3}. \quad (2)$$

Step 3: Let the leptons $L_{iL} = (\nu, l)_{iL} \sim \underline{1}, \underline{1}', \underline{1}''$, $l_{iR} \sim \underline{3}$, with $\Phi = (\phi^+, \phi^0) \sim \underline{1}$, then the usual SM Yukawa couplings $\bar{L}_{iL} l_{jR} \Phi$ are forbidden. Similarly, the quarks $Q_{iL} = (u, d)_{iL} \sim \underline{1}, \underline{1}', \underline{1}''$, $u_{iR} \sim \underline{3}$, $d_{iR} \sim \underline{3}$, thus also forbidding $\bar{Q}_{iL} d_{jR} \Phi$ and $\bar{Q}_{iL} u_{jR} \tilde{\Phi}$, where $\tilde{\Phi} = (\bar{\phi}^0, -\phi^-)$. The nonzero vacuum expectation value of ϕ^0 generates W and Z masses but not fermion masses.

Step 4: Consider first the charged leptons with

$$(\eta^+, \eta^0), \chi^+ \sim \underline{1}, \quad N_{iL} \sim \underline{3}, \quad N_{iR} \sim \underline{1}, \underline{1}', \underline{1}''. \quad (3)$$

Hence the Yukawa couplings $\bar{N}_R l_L \eta^+$ and $\bar{l}_R N_L \chi^-$ are allowed. The soft breaking of A_4 to Z_3 [8] in the 3×3 Dirac mass matrix of $N_{1,2,3}$, i.e.

$$\mathcal{M}_N = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{pmatrix}, \quad (4)$$

where $\omega = \exp(2\pi i/3)$, then allows Φ to couple to $\bar{l}_L l_R$ in one loop as shown in Fig. 1.

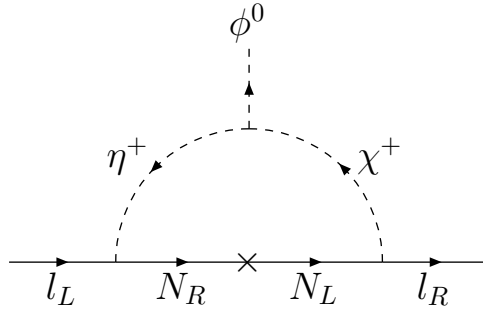


Figure 1: One-loop generation of charged-lepton mass.

Similar diagrams exist for the quarks, using the other scalars of Eq. (1).

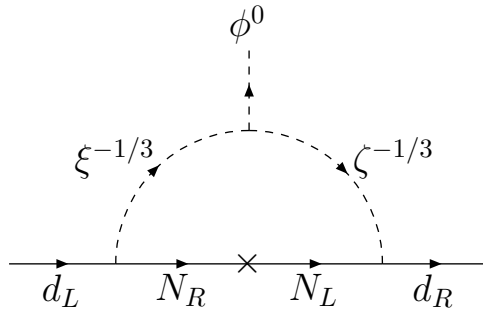


Figure 2: One-loop generation of d quark mass.

Thus all quarks and leptons owe their masses to dark matter in conjunction with Φ . Note that the DM particles are distinguished from the SM particles by Z_2 which may itself be a remnant of an $U(1)_D$ gauge symmetry.

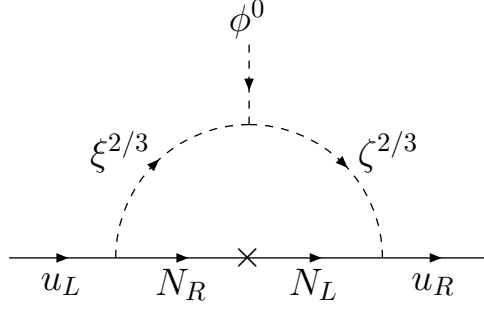


Figure 3: One-loop generation of u quark mass.

Step 5: The residual Z_3 symmetry maintains diagonal mass matrices for u and d quarks as well as charged leptons. This is an explanation of why the quark mixing matrix V_{CKM} is nearly diagonal.

Step 6: Further soft breaking of Z_3 allows a realistic quark mixing matrix (V_{CKM}). The $\bar{q}_{iL}q_{jR}$ mass matrix is of the form

$$\mathcal{M}_q = \begin{pmatrix} f_1 & 0 & 0 \\ 0 & f_2 & 0 \\ 0 & 0 & f_3 \end{pmatrix} U_M^L \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{pmatrix} (U_M^R)^\dagger. \quad (5)$$

If the unitary matrices $U_M^{L,R}$ are the identity, then Z_3 is not broken and V_{CKM} is also the identity. Let U_M^L be approximately given by

$$U_M^L \simeq \begin{pmatrix} 1 & -\epsilon_{12} & -\epsilon_{13} \\ \epsilon_{12}^* & 1 & -\epsilon_{23} \\ \epsilon_{13}^* & \epsilon_{23}^* & 1 \end{pmatrix} \quad (6)$$

then

$$\mathcal{M}_q \mathcal{M}_q^\dagger \simeq \begin{pmatrix} f_1^2 M_1^2 & f_1 f_2 \epsilon_{12} (M_1^2 - M_2^2) & f_1 f_3 \epsilon_{13} (M_1^2 - M_3^2) \\ f_1 f_2 \epsilon_{12}^* (M_1^2 - M_2^2) & f_2^2 M_2^2 & f_2 f_3 \epsilon_{23} (M_2^2 - M_3^2) \\ f_1 f_3 \epsilon_{13}^* (M_1^2 - M_3^2) & f_2 f_3 \epsilon_{23}^* (M_2^2 - M_3^2) & f_3^2 M_3^2 \end{pmatrix}. \quad (7)$$

Let $m_d \simeq f_1^d M_1$, $m_s \simeq f_2^d M_2$, $m_b \simeq f_3^d M_3$, $m_u \simeq f_1^u M_1$, $m_c \simeq f_2^u M_2$, $m_t \simeq f_3^u M_3$, then

V_{CKM} is approximately given by

$$V_{ud} \simeq V_{cs} \simeq V_{tb} \simeq 1, \quad V_{us} \simeq \left(\frac{m_d}{m_s} - \frac{m_u}{m_c} \right) \epsilon_{12} \left(\frac{M_2^2 - M_1^2}{M_2 M_1} \right), \quad (8)$$

$$V_{ub} \simeq \left(\frac{m_d}{m_b} - \frac{m_u}{m_t} \right) \epsilon_{13} \left(\frac{M_3^2 - M_1^2}{M_3 M_1} \right), \quad V_{cb} \simeq \left(\frac{m_s}{m_b} - \frac{m_c}{m_t} \right) \epsilon_{23} \left(\frac{M_3^2 - M_2^2}{M_3 M_2} \right). \quad (9)$$

There are many realistic solutions of the above. The simplest is to set $f_1^d = f_2^d = f_3^d$, in which case $V_{CKM} \simeq (U_M^L)^\dagger$. In other words, the soft breaking of Z_3 which generates U_M^L is directly linked to the observed V_{CKM} .

Step 7: As for the radiative generation of Majorana neutrino mass, the well-studied one-loop scotogenic model [9] (with Z_2) may be used. If $U(1)_D$ is desired, then the recent one-loop proposal [10] with two scalar doublets $(\eta_{1,2}^+, \eta_{1,2}^0)$ transforming oppositely under $U(1)_D$ is a good simple choice. However, a two-loop realization may also be adopted, as shown in Fig. 4, which may preserve $U(1)_D$ as well. Under Z_3 , $\nu_{e,\mu,\tau}, N_{e,\mu,\tau}, \rho_{1,2,3} \sim 1, \omega, \omega^2$, $(\phi^+, \phi^0), (\eta^+, \eta^0), \chi^0 \sim 1$. Under $U(1)_D$, $N, (\eta^+, \eta^0), \chi^0 \sim 1, \rho \sim 2$.

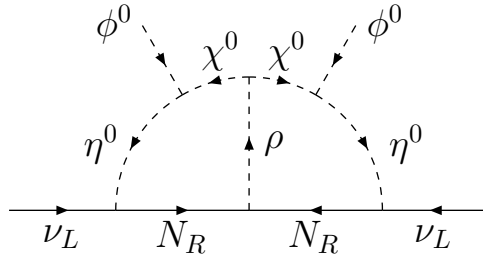


Figure 4: Two-loop generation of Majorana neutrino mass with $U(1)_D$ symmetry.

The addition of χ^0 and ρ_1 completes the two loops without breaking $U(1)_D$ or Z_3 . This would result in a Majorana neutrino mass matrix in the basis $(\nu_e, \nu_\mu, \nu_\tau)$ of the form

$$\mathcal{M}_\nu = \begin{pmatrix} A & 0 & 0 \\ 0 & 0 & B \\ 0 & B & 0 \end{pmatrix}. \quad (10)$$

The further addition of $\rho_{2,3}$ together with the soft breaking of Z_3 using the trilinear $\chi^0 \chi^0 \rho_{2,3}^\dagger$ couplings allows \mathcal{M}_ν to become

$$\mathcal{M}_\nu = \begin{pmatrix} A & C & C^* \\ C & D^* & B \\ C^* & B & D \end{pmatrix}, \quad (11)$$

where A and B are real. Note that this pattern is protected by a symmetry first pointed out in Ref. [11], i.e. $e \rightarrow e$ and $\mu - \tau$ interchange with CP conjugation, and obtained previously in Ref. [6]. As such, it is also guaranteed to yield maximal $\nu_\mu - \nu_\tau$ mixing ($\theta_{23} = \pi/4$) and maximal CP violation, i.e. $\exp(-i\delta) = \pm i$, whereas θ_{13} may be nonzero and arbitrary.

The mass matrix of Eq. (11) has six parameters: A, B, C_R, C_I, D_R, D_I , but only five are independent because the relative phase of C and D is unobservable. Using the conventional parametrization of the neutrino mixing matrix, the angle θ_{13} is given by

$$\frac{s_{13}}{c_{13}} = \frac{-D_I}{\sqrt{2}C_R}, \quad \frac{s_{13}c_{13}}{c_{13}^2 - s_{13}^2} = \frac{\sqrt{2}C_I}{A - B + D_R}. \quad (12)$$

The adjustable relative phase of C and D is used to allow the above two equations to be satisfied with a single θ_{13} . The angle θ_{12} is then given by

$$\frac{s_{12}c_{12}}{c_{12}^2 - s_{12}^2} = \frac{-\sqrt{2}(c_{13}^2 - s_{13}^2)C_R}{c_{13}[c_{13}^2(A - B - D_R) + 2s_{13}^2D_R]}. \quad (13)$$

As a result, the three mass eigenvalues are

$$m_1 = \frac{c_{13}^2[c_{12}^2A - s_{12}^2B - s_{12}^2D_R] - s_{13}^2[(c_{12}^2 - s_{12}^2)B - D_R]}{(c_{13}^2 - s_{13}^2)(c_{12}^2 - s_{12}^2)}, \quad (14)$$

$$m_2 = \frac{c_{13}^2[-s_{12}^2A + c_{12}^2B + c_{12}^2D_R] - s_{13}^2[(c_{12}^2 - s_{12}^2)B + D_R]}{(c_{13}^2 - s_{13}^2)(c_{12}^2 - s_{12}^2)}, \quad (15)$$

$$m_3 = \frac{s_{13}^2A - c_{13}^2B + c_{13}^2D_R}{c_{13}^2 - s_{13}^2}. \quad (16)$$

Since $s_{13}^2 \simeq 0.025$ is small, these expressions become

$$m_2 + m_1 \simeq A + B + D_R + s_{13}^2(A - B + D_R), \quad (17)$$

$$(c_{12}^2 - s_{12}^2)(m_2 - m_1) \simeq -A + B + D_R - s_{13}^2(A - B + D_R), \quad (18)$$

$$m_3 \simeq -B + D_R + s_{13}^2(A - B + D_R). \quad (19)$$

It is clear that a realistic neutrino mass spectrum with $m_2^2 - m_1^2 \ll |m_3^2 - (m_2^2 + m_1^2)/2|$ may be obtained with either $|m_1| < |m_2| < |m_3|$ (normal ordering) or $|m_3| < |m_1| < |m_2|$ (inverted ordering).

Step 8: The predicted scalars of Eq. (1) which connect the quarks and leptons to their common dark-matter antecedents, i.e. $N_{1,2,3}$, are possibly observable at the LHC. They may also change significantly the SM couplings of Φ [12]. In Fig. 1, η^+ is part of an electroweak doublet (η^+, η^0) and χ^+ is a singlet. They mix because of the $\mu(\eta^+\phi^0 - \eta^0\phi^+)\chi^-$ trilinear interaction. The 2×2 mass-squared matrix spanning (η^\pm, χ^\pm) is given by

$$\mathcal{M}_{\eta\chi}^2 = \begin{pmatrix} m_\eta^2 & \mu v/\sqrt{2} \\ \mu v/\sqrt{2} & m_\chi^2 \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}, \quad (20)$$

where $\langle\phi^0\rangle = v/\sqrt{2}$. Now $\zeta_1 = \eta \cos\theta + \chi \sin\theta$, $\zeta_2 = \chi \cos\theta - \eta \sin\theta$ are the mass eigenstates, and the mixing angle θ is given by

$$\frac{\mu v}{\sqrt{2}} = \sin\theta \cos\theta (m_1^2 - m_2^2). \quad (21)$$

The exact calculation of m_l in terms of the exchange of $\zeta_{1,2}$ results in [9, 10]

$$\begin{aligned} m_l &= \frac{f_\eta f_\chi \sin\theta \cos\theta}{16\pi^2} m_N \left[\frac{m_1^2}{m_1^2 - m_N^2} \ln \frac{m_1^2}{m_N^2} - \frac{m_2^2}{m_2^2 - m_N^2} \ln \frac{m_2^2}{m_N^2} \right] \\ &= \frac{f_\eta f_\chi \mu v}{16\sqrt{2}\pi^2(m_1^2 - m_2^2)} m_N \left[\frac{m_1^2}{m_1^2 - m_N^2} \ln \frac{m_1^2}{m_N^2} - \frac{m_2^2}{m_2^2 - m_N^2} \ln \frac{m_2^2}{m_N^2} \right]. \end{aligned} \quad (22)$$

Let $\phi^0 = (v + h)/\sqrt{2}$ and consider the effective Yukawa coupling $h\bar{l}l$. In the SM, it is of course equal to m_l/v , but here it has three contributions. Assuming that m_h^2 is small compared to $m_{1,2}^2$ and m_N^2 , and defining $x_{1,2} = m_{1,2}^2/m_N^2$, Fig. 1 yields

$$f_l^{(3)} = \frac{f_\eta f_\chi \mu}{16\sqrt{2}\pi^2 m_N} [(\cos^4\theta + \sin^4\theta)F(x_1, x_2) + \sin^2\theta \cos^2\theta (F(x_1, x_1) + F(x_2, x_2))], \quad (23)$$

$$F(x_1, x_2) = \frac{1}{x_1 - x_2} \left[\frac{x_1}{x_1 - 1} \ln x_1 - \frac{x_2}{x_2 - 1} \ln x_2 \right], \quad F(x, x) = \frac{1}{x - 1} - \frac{\ln x}{(x - 1)^2}. \quad (24)$$

Comparing Eq. (23) with Eq. (22), we see that $f_l^{(3)} = m_l/v$ only in the limit $\theta \rightarrow 0$. We see also that $F(x_1, x_1) + F(x_2, x_2)$ is always greater than $2F(x_1, x_2)$, so that $f_l^{(3)}$ is always greater than m_l/v . The correction due to nonzero m_h is easily computed in the limit $m_1 = m_2 = m_N$, in which case it is $m_h^2/12m_N^2$. This shows that it should be generally negligible. Let

$$F_+(x_1, x_2) = \frac{F(x_1, x_1) + F(x_2, x_2)}{2F(x_1, x_2)} - 1, \quad F_-(x_1, x_2) = \frac{F(x_1, x_1) - F(x_2, x_2)}{2F(x_1, x_2)}, \quad (25)$$

then $F_+ \geq 0$ and if $x_1 = x_2$, $F_+ = F_- = 0$. The other two contributions to $h\bar{\tau}\tau$ come from $\lambda_\eta v\eta^+\eta^-$ and $\lambda_\chi v\chi^+\chi^-$, i.e.

$$f_\tau^{(1)} = \frac{\lambda_\eta v f_\eta f_\chi}{16\pi^2 m_N} \sin\theta \cos\theta [\cos^2\theta F(x_1, x_1) - \sin^2\theta F(x_2, x_2) - \cos 2\theta F(x_1, x_2)], \quad (26)$$

$$f_\tau^{(2)} = \frac{\lambda_\chi v f_\eta f_\chi}{16\pi^2 m_N} \sin\theta \cos\theta [\sin^2\theta F(x_1, x_1) - \cos^2\theta F(x_2, x_2) + \cos 2\theta F(x_1, x_2)]. \quad (27)$$

Let $r_{\eta,\chi} = \lambda_{\eta,\chi} v^2 / m_N^2$, then the total contribution to the Higgs Yukawa coupling is given by

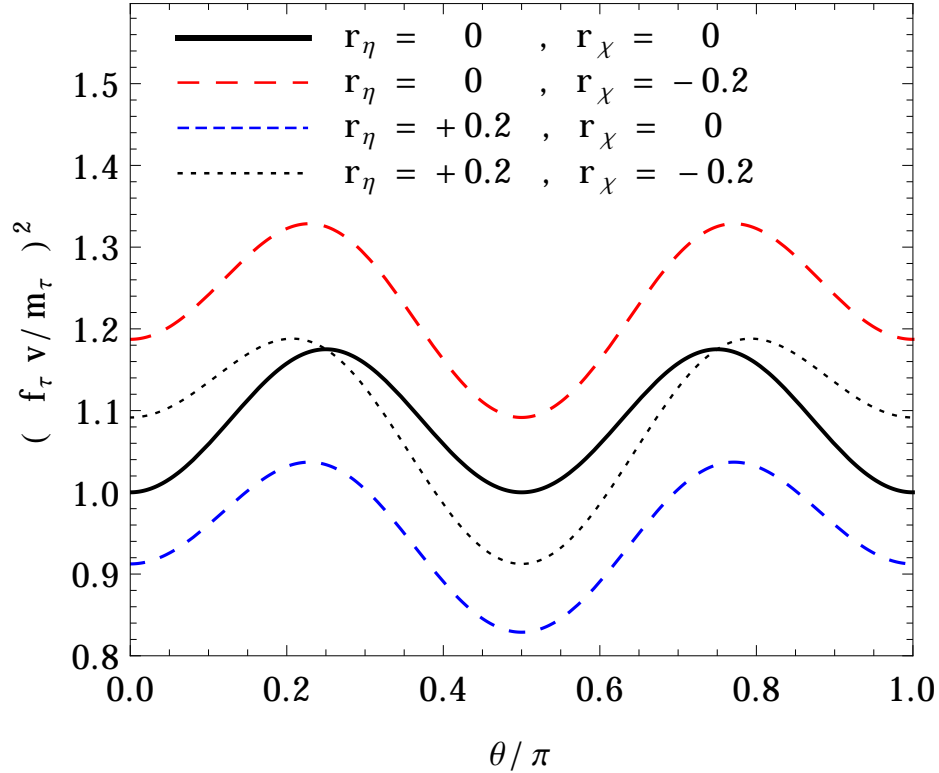


Figure 5: The ratio $(f_\tau v / m_\tau)^2$ plotted against θ for $x = 2$ and various r_η and r_χ .

$$\frac{f_\tau v}{m_\tau} = 1 + \left[\frac{1}{2} (\sin 2\theta)^2 + \frac{1}{4} \sin 4\theta (r_\eta - r_\chi) \right] F_+ + \frac{1}{2} \sin 2\theta (r_\eta + r_\chi) F_-. \quad (28)$$

The LHC measurements of $h \rightarrow \tau^+\tau^-$ and $h \rightarrow b\bar{b}$ provide the bounds

$$\left(\frac{f_\tau v}{m_\tau} \right)^2 = 1.4 \begin{pmatrix} +0.5 \\ -0.4 \end{pmatrix}, \quad \left(\frac{f_b v}{m_b} \right)^2 = 0.2 \begin{pmatrix} +0.7 \\ -0.6 \end{pmatrix} \quad (\text{ATLAS}) [13], \quad (29)$$

$$\left(\frac{f_\tau v}{m_\tau} \right)^2 = 1.10 \pm 0.41, \quad \left(\frac{f_b v}{m_b} \right)^2 = 1.15 \pm 0.62 \quad (\text{CMS}) [14]. \quad (30)$$

The ratio $(f_\tau v/m_\tau)^2$ is plotted against θ for $x = 3$ and various r_η and r_χ in Fig. 5.

In conclusion, the 126 GeV particle may hold secrets of physics beyond the SM. It could indeed be the one Higgs, but not exactly that of the SM. Its couplings to fermions may hold the key to understanding flavor and dark matter. Particles which look like scalar quarks and leptons are also predicted at the LHC, but with properties not exactly like those required by supersymmetry. The Higgs Yukawa couplings to fermions may also differ significantly from the SM predictions.

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